An Adverse Selection Model with Finite Number of Types and Informational Rents

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Abstract. In the paper we analyze a contractual relationship between two economic agents using a standard Principal-Agent approach from the theory of incentives. The Principal wants to delegate a production activity to an Agent privately informed about his marginal cost of production. This problem corresponds to a classical model of adverse selection. We first present the standard model with two types of agent and then we generalize it, assuming that the type of agent belongs to a finite set of values. We provide a full characterization of the Principal’s optimization problem and we solve it using Kuhn-Tucker techniques. Focusing on the economic interpretation of the optimal solution, we also use a change of variables. The optimization problem is next expressed in terms of the new variables, the informational rents. In the last part of the paper we derive and summarize the characteristics of the optimal contracts in the situation of asymmetric information.

Key words: Pareto efficiency, adverse selection model, optimal incentive contracts
JEL classification: C61, D82

1 Introduction

In the paper we consider a classical Principal-Agent problem from the incentives theory, in a static bilateral contracting framework. One of the participants, the Agent, is better informed with respect to some of his characteristics and this private information affects the contract’ results. The other partner named the Principal wants to delegate a production activity, but can not perfectly observe or verify the Agent’s private information. The only way to control the Agent’s decisions is to design and to propose to the Agent an optimal incentive contract. This contract is derived in such manner that at the optimum the allocation of the resources is efficient when the Principal gives up to an informational rent to the better informed Agent.

This delegation problem corresponds to a classical adverse selection problem where the information gap between the Principal and the Agent has strong implications for the design of optimal contract to be signed.

The term adverse selection was first used in insurances, describing the situation when the individual demand for the insurance is positively correlated with the individual risk of loss. Lately, the literature reffering the adverse selection (anti-selection or negative selection) models is very vast. While in theory the adverse selection seems to be the consequences of the economic incentives, empirical evidences are mixed (Cawley and Philipson, 1999, Chiappori and Salanie, 2000, Finkelstein and McGarry, 2006).


Recently, in the labour market literature we may find the dynamic adverse selection model
produced and \( t \) – the transfer received by the Agent (the Principal’s payment to the Agent if the later produces \( q \) units for the Principal). So, the Principal’s objective is to design and propose to the Agent a pair \( (q, t) \).

**A4.** The timing of contracting in adverse selection situation (in asymmetric information): \( t = 0 \): the Agent discovers his type \( \theta \); but the Principal does not observe this type; 
\( t = 1 \): the Principal design and offers the contract; 
\( t = 2 \): the Agent accepts or refuses the contract; 
\( t = 3 \): the contract is executed; payoffs for Principal and Agent.

### 2.1 The optimal contract in the situation of symmetric information

First we suppose that the Principal knows exactly the Agent’s type such that there is no asymmetry of information between the Principal and the Agent. The contractual offer the Principal proposes to the Agent corresponds to the solution of the following optimization problem:

\[
\left( \max \right) \{ S(q) - t \}
\]

\[ s.t. \]
\[ t \geq \theta q \]
\[ q \geq 0, t \geq 0 \]

The first order conditions for the above problem are also sufficient conditions, the objective function being concave. So, we have:

\[
(FOC) \quad \left\{ \begin{array}{l}
S'(q^*) = \theta \Rightarrow q^* = (S')^{-1}(\theta) \\
t^* = \theta q^*
\end{array} \right.
\]

Note that the first best contract yields to a higher optimal production for the efficient type (the type of Agent with low marginal costs) than the corresponding production for the inefficient type (the type of Agent with high marginal costs), i.e. \( q^*_1 > q^*_2 \).

### 2.2 The case of asymmetric information

Suppose now that the asymmetric information

\[ \text{Suppose now that the Principal does not know the Agent’s type, but he has some beliefs} \]
regarding the Agent’s type: we denote by $\nu$ the probability that the Agent is efficient ($\vartheta$); then, the probability that the Agent is inefficient ($\bar{\vartheta}$) is $1-\nu$.

In this situation, it is optimal for the Principal to design a menu of contracts – one for each type, hoping that each type of Agent chooses the contract designed for him. We denote by $\{(t,q),(\bar{t},\bar{q})\}$ the menu of contracts being derived such that the Principal’s expected profit is maximized:

$$\max_{\tau,\pi,\underline{q}} \left[ \nu S(q) - \tau + (1-\nu)S(\bar{q}) - \bar{\tau} \right]$$

The optimal contracts are chosen from the set of the incentive feasible contracts, meaning the contracts satisfying the participation constraints:

$$t - \bar{\vartheta} q \geq 0 \quad (1)$$
$$\bar{t} - \vartheta \bar{q} \geq 0 \quad (2)$$

and the incentive compatibility constraints:

$$t - \bar{\vartheta} q \geq \bar{t} - \vartheta \bar{q} \quad (3)$$
$$\bar{t} - \vartheta \bar{q} \geq t - \bar{\vartheta} q \quad (4)$$

We will not insist on deriving the optimal solution of the above problem. The interested reader can find all the details in Laffont and Martimort (Laffont and Martimort, 2002). We only present here the features of the optimal contracts in the case of adverse selection with two types of Agent:

i) The Agent with low marginal cost marginal (the efficient Agent) produces the first best quantity, with

$$S'(q) = \vartheta \quad (5)$$

ii) The Agent with high marginal cost (the inefficient Agent) produces a distorted quantity with respect to the first best, and this quantity is given by the following relation:

$$S'(q^{SB}) = \vartheta + \Delta \vartheta \frac{\nu}{1-\nu} \quad (6)$$

where $\Delta \vartheta = \bar{\vartheta} - \vartheta$, and $q^{SB} < \bar{q}$. iii) The Agent with type $\bar{\vartheta}$ gets no informational rent (hence he obtains exactly his outside opportunity level of utility). We have therefore $\bar{U}^{SB} = 0$ or his optimal transfer is:

$$\bar{t}^{SB} = \bar{\vartheta} q^{SB} \quad (7)$$

iv) The efficient Agent gets a positive informational rent, given by $U^{SB} = \Delta \vartheta q^{SB}$ or his optimal transfer is:

$$t^{SB} = \vartheta q^{SB} + \Delta \vartheta q^{SB} \quad (8)$$

3 A generalization of the adverse selection model: finite number of types

In this section we use the above standard model and we extend it, modifying the assumption regarding the Agent’s private information. We now suppose that the Agent can have one of $n$ types (marginal costs): $\vartheta_1 < \vartheta_2 < \vartheta_3 < ... < \vartheta_n$, where $\vartheta_i$ represents the most efficient type (the type with the lowest marginal cost), and $\vartheta_e$ is the least efficient type (the type with the highest marginal cost).

The probability that the Agent has the type $\vartheta_k$ is denoted by $\nu_k > 0$, with $\sum_{i=1}^{n} \nu_i = 1$.

3.1 The menu of incentive feasible contracts and the Principal’s optimization problem

We define an incentive feasible menu of contracts $\{(t_i,q_i),(t_2,q_2),...,(t_n,q_n)\}$ if it satisfies the following constraints:

- participation constraints:

$$RP \quad t_i - \vartheta_i q_i \geq 0, \quad i = 1, n \quad (9)$$

- incentive compatibility constraints:

$$RCI \quad t_i - \vartheta_i q_i \geq t_j - \vartheta_j q_j, \quad \forall i, j = 1, n, i \neq j \quad (10)$$

The objective function of the Principal and the optimization problem

The decisional problem of the Principal consists in designing an optimal contract he proposes to the Agent. This contract must be chosen from the set of incentive feasible contracts and it maximizes the Principal’s expected profit. Hence, the Principal’s optimization problem is written as:
\[
\left( \max_{(q_i,q_j)} \right) \sum_{i=1}^{n} v_i [S(q_i) - t_i] \\
\text{s.t.} \quad (RP_i) \quad t_i - \theta_i q_i \geq 0, \quad i = 1, n \\
(RCI_j) \quad t_j - \theta_j q_j \geq t_j - \theta q_j, \quad \forall i, j = 1, n, i \neq j
\]

Bolton and Dewatripont (Bolton and Dewatripont, 2005) proposed in their book a real guide of solving such a complex model of adverse selection. Next, we will follow their procedure.

3.2 Reducing the dimension of the Principal’s problem

The main difficulty in solving the above problem is to reduce the number of the constraints: we have \( n + n(n - 1) \) constraints, i.e. \( n \) participation constraints and \( n(n - 1) \) incentive compatibility constraints. In order to reduce the number of the relevant constraints we follow some steps (given in the following propositions):

First, note that from all the participation constraints, the only relevant constraint is the constraint assigned to the least efficient type (the type with the highest marginal costs) \( \theta_n \).

This remark follows immediately from \( (RCI_{in}) \) and \( (RP_i) \). These relations yield together to:

\[
t_i - \theta_i q_i \geq t_n - \theta_n q_n \geq t_n - \theta q_n \geq 0 \quad (11)
\]

**Proposition 1.** If the set of incentive feasible contracts is nonempty, then the implementability condition holds.

**Remark:** The Spence-Mirrlees condition (the single crossing condition) is satisfied because we have

\[
\frac{\partial}{\partial \theta} \left[ -\frac{\partial U}{\partial q} \frac{\partial q}{\partial t} \right] > 0,
\]

since

\[
\frac{\partial}{\partial \theta} \left[ -\frac{\theta}{1} \right] = 1 > 0.
\]

Summing the constraints \( (RCI_j) \) and \( (RCI_{in}) \) we get

\[
\theta_i (q_j - q_i) + \theta_j (q_i - q_j) \geq 0 \quad (12)
\]

or

\[
(\theta_i - \theta_j) (q_j - q_i) \geq 0 \quad (13)
\]

Therefore, whenever \( \theta_i > \theta_j \), it must be that \( q_j \geq q_i \). If the sequence \( \{\theta_i\} \) is increasing, then the sequence is \( \{q_i\} \) decreasing.

**Proposition 2.** Suppose that \( \theta_{r-1} < \theta_i < \theta_{r+1} \) and that the local upward incentive constraints

\[
t_{r-1} - \theta_{r-1} q_{r-1} \geq t_{r-1} - \theta_{r-1} q_{r-1} + t_{r-1} + t_{r+1} \quad (14)
\]

or

\[
t_{r-1} - \theta_{r-1} q_{r-1} \geq t_{r-1} q_{r-1} - \theta_{r-1} q_{r+1} + t_{r+1} \quad (15)
\]

Since \( \theta_i > \theta_{r-1} \), we have:

\[
t_{r-1} - \theta_{r-1} q_{r-1} \geq \theta_{r-1} \left[ q_i - q_{r-1} \right] - \theta_{r-1} q_{r+1} + t_{r+1} \quad (16)
\]

or

\[
t_{r-1} - \theta_{r-1} q_{r-1} \geq t_{r-1} - \theta_{r-1} q_{r+1} \quad (17)
\]

We can easily interpret the above proposition: if for each type \( \theta_i \) the incentive constraint with respect to type \( \theta_{r-1} \) holds, then all other upward incentive constraints (for \( \theta_i \) relative to higher types) are also satisfied if the implementability condition holds. One can similarly show that the set of downward incentive constraints (for \( \theta_i \) relative to lower types) is also satisfied.

We can conclude that the set of \( n(n-1) \) constraints can be replaced by the set of local incentive constraints together with the implementability condition and the participation constraint of the type \( \theta_n \).

The next question (step) is whether the set of remaining constraints can be reduced further and to establish (show) which constraints are binding at the optimum.

**Proposition 3.** At the optimum, all the local upward constraints are binding.

If this is not true, suppose that the constraint \( (RCI_{r+1}) \) is not binding, that is i.e.:

\[
t_i - \theta_i q_i > t_{r+1} - \theta_i q_{r+1} \quad (18)
\]

In this case the Principal could reduce the transfer \( t_i \) (given to the Agent with type \( \theta_i \))

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and all other transfers $t_j$ (with $j < i$) by a positive amount $\varepsilon > 0$ so as to make the preceding constraints binding. Doing so, the Principal gets a profit surplus given by

$$\varepsilon \left( \sum_{j \neq i} v_j \right).$$

We can easily show that the only remaining participation constraint is also binding at the optimum. Hence, we have:

$$t_n - \theta_n q_n = 0 \quad (19)$$

**Proposition 4.** If all the local upward constraints are binding at the optimum, then all the downward incentive constraints are satisfied.

That is: if $t_i - \theta_i q_i = t_{i+1} - \theta_i q_{i+1}$, then we have

$$t_i - \theta_i q_i \geq t_{i-1} - \theta_i q_{i-1}.$$

**Proof**

We use the local upward constraint for the index $i-1$:

$$t_{i-1} - \theta_{i-1} q_{i-1} = t_i - \theta_{i-1} q_i \quad (20)$$

It follows that:

$$t_{i-1} = \theta_{i-1} q_{i-1} + t_i - \theta_{i-1} q_i \quad (21)$$

We now have:

$$t_{i-1} - \theta_{i-1} q_{i-1} - \theta_{i-1} q_i + t_{i+1} - \theta_{i+1} q_{i+1} \leq t_i + \theta_i (q_{i-1} - q_i) - \theta_{i-1} q_{i-1} = t_i - \theta_i q_i \quad (22)$$

### 4 Solving the reduced problem of the Principal

With all the results from the previous section, the problem is significantly reduced and we can now solve this final form of the Principal’s problem having the participation constraint and the incentive constraints binding at the optimum and with the implementability condition.

The Principal’s problem reduces to:

\[
\begin{align*}
(P_{\text{reduced}}): & \max_{(q_i, t_i)} \sum_{i=1}^{n} v_i \left[ S(q_i) - t_i \right] \\
\text{s.t.} & \quad (RP_n) \quad t_n - \theta_n q_n = 0 \\
& \quad (RCI_{i+1}) \quad t_i - \theta_i q_i = t_{i+1} - \theta_i q_{i+1}, \quad \forall i = 1, n-1 \\
& \quad (C Im) \quad q_1 \geq q_2 \geq \ldots
\end{align*}
\]

#### 4.1 Solving the relaxed problem-ignoring the implementability condition

In solving the problem, we use the same logic as in the standard model of adverse selection. We ignore for a while the last condition-the implementability condition (CIm). At the final, in the next section, we analyze if the solution we previous derived satisfies this implementability condition.

We assign the multiplier denoted by $\mu$ to the participation constraint and the multipliers denoted by $\lambda_i, i = 1, n-1$ to the incentive constraints. The Lagrangian is written as:

$$L = \sum_{i=1}^{n} v_i \left[ S(q_i) - t_i \right] + \mu [t_n - \theta_n q_n] + \sum_{i=1}^{n} \lambda_i (t_i - \theta_i q_i - t_{i+1} + \theta_i q_{i+1})$$

Assuming that the Principal does not want to eliminate the least efficient type (that is $i.e. q_n > 0$) then all other quantities produced at the optimum satisfy the relation $q_i > 0$.

From the participation constraints it follows immediately that all the optimal transfers must satisfy $t_i > 0$.

The first order conditions are:

$$\frac{\partial L}{\partial q_i} = 0 \quad (i = 2, n-1) \quad \text{or}$$

$$v_i S'(q_i) - \lambda_i \theta_i + \lambda_{i-1} \theta_{i-1} = 0, \quad i = 2, n-1 \quad (23)$$

$$\frac{\partial L}{\partial t_i} = 0 \quad (i = 2, n-1) \quad \text{or}$$

$$-v_i + \lambda_i - \lambda_{i-1} = 0, \quad i = 2, n-1 \quad (24)$$

$$\frac{\partial L}{\partial q_i} = 0 \quad \Rightarrow \quad -v_i + \lambda_i = 0 \quad \Rightarrow \quad \lambda_i = v_i > 0 \quad (25)$$

$$\frac{\partial L}{\partial q_n} = 0 \quad \Rightarrow \quad v_i S'(q_i) - \lambda_i \theta_i = 0 \quad (26)$$

$$\frac{\partial L}{\partial q_n} = 0 \quad \Rightarrow \quad v_i S'(q_n) - \mu \theta_i + \lambda_{n-1} \theta_{n-1} = 0 \quad (27)$$

$$\frac{\partial L}{\partial t_n} = 0 \quad \Rightarrow \quad \mu - \lambda_{n-1} - v_n = 0 \quad \Rightarrow \quad \mu = \lambda_{n-1} + v_n > 0 \quad (28)$$

Using these first order conditions we derive the characteristics of the optimal contracts (the second best solution).
From (24) we get $\lambda_i - \lambda_{i-1} = v_i, i = 2, n - 1$ and summing all these relations we obtain:

$$\lambda_j - \lambda_1 = v_2 + v_3 + ... + v_j$$

(29)

But from (25) we have $\lambda_1 = v_1$, and this implies:

$$\lambda_j = \sum_{i=1}^{j} v_i > 0$$

(30)

The condition (26) gives the following equation:

$$S'(q_i) = \theta_i \Rightarrow q_i^{SB} = q_i^*$$

(31)

(the production of the Agent with type $\theta_i$ is efficient, meaning that it corresponds to the first best solution).

From (23) we also have:

$$v_j S'(q_i) = \lambda_j \theta_i - \lambda_{j-1} \theta_{j-1} = v_j \theta_i + \lambda_{j-1} (\theta_i - \theta_{j-1})$$

or,

$$S'(q_i) = \theta_i + \frac{\theta_i - \theta_{i-1}}{v_i} \sum_{j=1}^{i} v_j$$

(32)

(33)

It is obvious that $S'(q_i) > \theta_i = S'(q^*_i)$ and then $q_i^{SB} < q_i^*, i = 2, n - 1$ (the optimal second best production is inefficient for all types $\theta_2, \theta_3, ..., \theta_{n-1}$).

From (27) and (28) we get:

$$\nu_i S'(q_i) = \mu \theta_i - \lambda_{i-1} \theta_{i-1} = \lambda_{i-1} (\theta_i - \theta_{i-1}) + \lambda_i \theta_i$$

or,

$$S'(q_i) = \theta_i + \frac{\theta_i - \theta_{i-1}}{\nu_i} \sum_{j=1}^{i} \nu_j$$

(34)

(35)

We also have $S'(q_i) > \theta_n = S'(q_n^*)$, and this implies $q_n^{SB} < q_n^*$.

All the incentive constraints being binding at the optimum, the optimal transfers can be easily derived. Hence:

From the binding participation constraint $(R_{PN})$, $t_i - \theta_n q_n = 0$, we get:

$$t_i^{SB} = \theta_n q_n^{SB}$$

(36)

Then, from each binding incentive constraint $(RCI_{i+1})$ it follows that:

$$t_i = \theta_i q_i + t_{i+1} - \theta_i q_{i+1}$$

(37)

And this yields to:

$$t_i^{SB} = \theta_i [q_i^{SB} - q_{i+1}^{SB}] + t_{i+1}^{SB}$$

(38)

with $i = 1, n - 1$.

These transfers must be derived from the highest index $i = n - 1$ to the lowest index $i = 1$.

4.2 Verifying the implementability condition

Having derived the solution of the relaxed program, it remains to verify if the solution we derived in the previous section satisfies the ignored implementability condition, i.e.:

$$q_i^{SB} \geq q_i^{SB}, \forall \theta_i = 1, n - 1.$$

This condition is equivalent to the next relation:

$$q_i^{SB} \geq q_{i-1}^{SB} \Leftrightarrow S'(q_i^{SB}) \leq S'(q_{i-1}^{SB})$$

(39)

or,

$$\theta_i + \frac{\theta_i - \theta_{i-1}}{v_i} \sum_{j=1}^{i} v_j \leq \theta_i + \frac{\theta_i - \theta_{i-1}}{v_{i-1}} \sum_{j=1}^{i-1} v_j$$

(40)

or,

$$\frac{\theta_i - \theta_{i-1}}{v_i} \sum_{j=1}^{i} v_j \leq \frac{\theta_i - \theta_{i-1}}{v_{i-1}} \sum_{j=1}^{i-1} v_j, \forall \theta_i = 1, n - 1$$

(41)

5 Introducing the informational rents

We are coming back now to the first form of the Principal’s optimization problem. We here provide an alternative way to solve this problem, using as variables the Agents’ informational rents and the quantities to be produced.

Let $U_i = t_i - \theta_i q_i$ be the informational rent of the Agent with type $i$.

With this change of variables, our goal is to transform the initial program of the Principal. Hence, we have:

- The participation constraints become simple sign constraints, i.e.:

$$U_i \geq 0, i = 1, 2, ..., n$$

(42)

- The local and global upward incentive constraints can be written as:

$$U_i = t_i - \theta_i q_i \geq t_i - \theta_i q_{i+1} = U_{i+1} + (\theta_{i+1} - \theta_i) q_{i+1}$$

(43)

Further, we assume (without losing generality) that the differences between the marginal costs of production are the same for all agents, such that:
\[ \Delta \theta = \theta_{i+1} - \theta_i, i = 1, 2, ..., n - 1 \] (44)
With this additional assumption, the upward incentive constraints become:
\[ U_i \geq U_j + (j-i) \Delta \theta q_j \] (45)
with \( i = 1, 2, ..., n-1, j = i+1, i+2, ..., n, i < j \).

Remark: If \( j-i = 1 \), the above expression corresponds to a local upward constraint. If \( j-i > 1 \), the constraints are global upward constraints.

- The local and global downward incentive constraints are written as:
\[ U_j = t_j - \theta_j q_j \geq t_i - \theta_i q_i = U_i - (j-i) \Delta \theta q_i \] (46)
with \( i = 1, 2, ..., n-1, j = i+1, i+2, ..., n, i < j \).

Remark: If \( j-i = 1 \), the above constraint corresponds to a local downward constraint. If \( j-i > 1 \), the constraints are global downward constraints.

We will follow a similar approach to that presented in the previous section. First, we can easily show that the implementability condition holds. Hence, adding the constraints given in (45) and (46) we get:
\[ q_i \geq q_j, \forall i, j, j > i, i = 1, 2, ..., n - 1 \] (47)
The last condition holds also for \( j = i+1 \), such that we have:
\[ q_i \geq q_{i+1}, i = 1, 2, ..., n - 1 \] (48)
meaning that:
\[ q_1 \geq q_2 \geq ... \geq q_n \] (49)
The second step consists in proving that the global upward constraints are implied by the local upward constraints. We use the following local constraints:
\[ U_i \geq U_{i+1} + \Delta \theta q_{i+1} \]
\[ U_{i+1} \geq U_{i+2} + \Delta \theta q_{i+2} \]
\[ \vdots \]
\[ U_{j-1} \geq U_j + \Delta \theta q_j \]
Adding the above constraints we get:
\[ U_i \geq U_j + \Delta \theta (q_{i+1} + q_{i+2} + \ldots) \]
\[ \geq U_j + \Delta \theta (q_j + q_j + \ldots) \]
or
\[ U_i \geq U_j + (j-i) \Delta \theta q_j \] (50)
And this is exactly the global upward constraint for the pair \((i, j)\).

Therefore, we can ignore the global upward constraints when solving the problem. We must consider only the following constraints:
\[ U_1 \geq U_2 + \Delta \theta q_2 \]
\[ U_2 \geq U_3 + \Delta \theta q_3 \]
\[ \vdots \]
\[ U_{n-1} \geq U_n + \Delta \theta q_n \]
This group of constraints is also useful in proving that only one participation constraint is relevant. Hence, it is easy to check the following statement: if \( U_n \geq 0 \), then \( U_i \geq 0, i = 1, 2, ..., n-1 \).

We can now write the Principal’s optimization problem expressed in terms of the new variables \( q_1, q_2, ..., q_n, U_1, U_2, ..., U_n \):
\[
\text{Max} \sum_{i=1}^{n} V_i \left[ S(q_i) - \theta_i q_i \right] - \sum_{i=1}^{n} \sum_{j=1}^{n} U_i
\text{st.}
U_1 \geq U_2 + \Delta \theta q_2
U_2 \geq U_3 + \Delta \theta q_3
\vdots
U_{n-1} \geq U_n + \Delta \theta q_n
U_n \geq 0
\]
Note that in the above program we ignored all the downward constraints. This is because it is more likely that an agent with the type \( \theta_j \) is not declaring having the type \( \theta_i \), where \( \theta_j > \theta_i \) (an agent with higher marginal costs is not willing to declare that he has low marginal costs). Later we will check that these constraints are really satisfied by the optimal solution.

**Proposition 5.** The \( n-1 \) local upward constraints and the participation constraint \( U_n \geq 0 \) from the optimization problem are binding at the optimum.

**Proof**
Suppose that \( U_n > 0 \) and let \( \epsilon > 0 \) be a small positive value such that \( U_n - \epsilon \geq 0 \). Then, we can reduce all the remaining informational rents such that the following inequality holds:
\[ U_i - \epsilon \geq U_{i+1} - \epsilon + \Delta \theta q_i, \forall i = 1, 2, ..., n-1 \] (51)
If the optimal solution is denoted by \( (q_1, q_2, ..., q_n, U_1, U_2, ..., U_n) \), then the solution
\((q_1, q_2, \ldots, q_n, U_1 - \varepsilon, U_2 - \varepsilon, \ldots, U_n - \varepsilon)\) is at least feasible solution. But reducing the informational rents of the agents, the Principal gets a higher profit, i.e. the value of the objective function is increased by \(\varepsilon > 0\). And this is a contradiction. It follows that, at the optimum we have \(U_n = 0\).

Similarly we can prove that each local upward constraint is binding at the optimum. Therefore, the optimal solution is such that:

\[
U_n = 0 \quad (52)
\]

\[
U_{n-1} = \Delta \theta q_n, U_{n-2} = \Delta \theta (q_n + q_{n-1}) \ldots,
\]

\[
U_1 = \Delta \theta (q_n + q_{n-1} + \ldots + q_2)
\]

Generalizing, we can write the binding constraints as:

\[
U_i = \Delta \theta \left( \sum_{p=i+1}^{n} q_p \right)
\]

(53)

In the following step, we can use the results from the Proposition 6 in order to reduce the optimization problem’s dimension. Substituting (53) into the objective function we get:

\[
\max \left\{ \sum_{i=1}^{n} v_i [S(q_i) - \theta q_i] - \sum_{i=1}^{n} \left( v_i \sum_{p=i+1}^{n} q_p \right) \Delta \theta \right\}
\]

We obtained a reduced program with fewer variables and without constraints. More than this, the objective function being concave, the first order conditions are also sufficient conditions.

We denote by \(F(q_1, q_2, \ldots, q_n)\) the new objective function. It can be rewritten as:

\[
F(q_1, q_2, \ldots, q_n) = \sum_{i=1}^{n} v_i \left[ S(q_i) - \theta q_i \right] - \sum_{i=1}^{n} v_i (q_{i+1} + q_{i+2} + \ldots + q_n) \Delta \theta
\]

The first order conditions are the following:

\[
\frac{\partial F(\cdot)}{\partial q_i} = 0 \quad \text{or} \quad v_i \left[ S'(q_i) - \theta \right] = 0 \quad (54)
\]

and, for each index \(i = 2, 3, \ldots, n\)

\[
\frac{\partial F(\cdot)}{\partial q_i} = 0 \quad \text{or} \quad v_i \left[ S'(q_i) - \theta \right] - (v_1 + v_2 + \ldots + v_{i-1}) \Delta \theta = 0 \quad (55)
\]

These conditions are similar with those found in Section 4, so the optimal solution has the characteristics derived there.

6 Conclusions

We analyzed above the solution of the Principal’s optimization problem. We can state now the main characteristics of the optimal contracts in the situation of asymmetric information. The following theorem summarizes the findings.

Theorem. If the adverse selection parameters (the marginal costs of production) satisfy the condition:

\[
\frac{\theta_{i-1} - \theta_i}{\theta_{i+1} - \theta_i} \leq \frac{\sum_{j=1}^{i} v_j}{\sum_{j=1}^{i+1} v_j}, \forall i = 1, n-1
\]

then the optimal contract is characterized by the following:

A. Optimal productions:

\(\rightarrow\) The Agent with the lowest marginal cost (the type \(\theta_i\)) produces efficient, such that the optimal second best quantity is exactly the quantity from the first best solution. We have therefore:

\[
S'(q_i) = \theta_i \Rightarrow q_i^{SB} = q_i^*.
\]

\(\rightarrow\) All other types of Agent produce optimal quantities distorted with respect to the first best solution. The second best productions are given by the equations:

\[
S'(q_j) = \theta_j + \frac{\theta_{j-1} - \theta_j}{\nu_j} \sum_{j=1}^{i+1} v_j, i = 2, n-1,
\]

with \(q_i^{SB} < q_i^*, i = 2, n-1\)

and

\[
S'(q_n) = \theta_n + \frac{\theta_{n-1} - \theta_n}{\nu_n} \sum_{j=1}^{n} v_j,
\]

with \(q_n^{SB} < q_n^*\).

B. Optimal transfers:

\(\rightarrow\) The Agent with the highest marginal cost of production (the type \(\theta_1\)) gets a transfer equal to his total cost of optimal; that is:

\[
t_n^{SB} = \theta_n q_n^{SB}
\]
It follows that this type of Agent gets no informational rent from his contractual relation with the Principal, such that his optimal utility is exactly his outside opportunity level of utility.

→ The other types of Agent get optimal second best transfers given by:

$$t_{i}^{SB} = \theta_{i} \left[ q_{i}^{SB} - q_{i+1}^{SB} \right] + t_{i+1}^{SB},$$

where $$i = 1, n-1$$.

C. Optimal informational rents

→ The Agent with the highest marginal cost of production (the type $$\theta_{n}$$) gets no informational rent from his contractual relation with the Principal, such that his optimal utility is exactly his outside opportunity level of utility. We have: $$U_{n} = 0$$.

→ The other types of Agent get positive optimal informational rents given by:

$$U_{i} = \Delta \theta \left( \sum_{p=i+1}^{n} q_{p}^{SB} \right),$$

where $$i = 1, 2, ..., n-1$$

We proposed in the paper a generalization of a standard model of adverse selection, assuming that the adverse selection parameter (here-the marginal cost of production) can have one value from a finite number of values. This assumption is more appropriate in many empirical settings where the private information cannot be structured in only two types (groups). We presented a detailed procedure of solving this extended model and at the end we derived the main characteristics of the optimal contracts in the situation of asymmetric information.

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